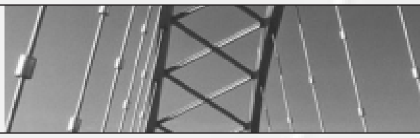


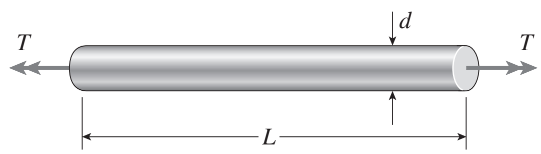
3

Torsion

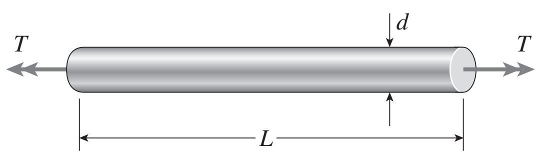
Torsional Deformations

Problem 3.2-1 A copper rod of length $L = 18.0$ in. is to be twisted by torques T (see figure) until the angle of rotation between the ends of the rod is 3.0°.

If the allowable shear strain in the copper is 0.0006 rad, what is the maximum permissible diameter of the rod?



Solution 3.2-1 **Copper rod in torsion**



$L = 18.0$ in.

$$\phi = 3.0° = (3.0)\left(\frac{\pi}{180}\right) \text{rad}$$

$$= 0.05236 \text{ rad}$$

$\gamma_{allow} = 0.0006$ rad

Find d_{max}

From Eq. (3-3):

$$\gamma_{max} = \frac{r\phi}{L} = \frac{d\phi}{2L}$$

$$d_{max} = \frac{2L\gamma_{allow}}{\phi} = \frac{(2)(18.0 \text{ in.})(0.0006 \text{ rad})}{0.05236 \text{ rad}}$$

$$d_{max} = 0.413 \text{ in.} \longleftarrow$$

Problem 3.2-2 A plastic bar of diameter $d = 50$ mm is to be twisted by torques T (see figure) until the angle of rotation between the ends of the bar is 5.0°.

If the allowable shear strain in the plastic is 0.012 rad, what is the minimum permissible length of the bar?

Solution 3.2-2 **Plastic bar in torsion**

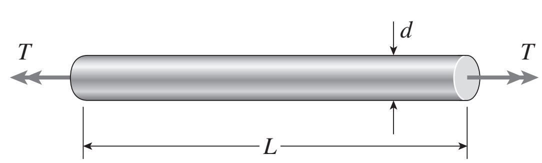
$d = 50$ mm

$$\phi = 5.0° = (5.0)\left(\frac{\pi}{180}\right) \text{rad} = 0.08727 \text{ rad}$$

$\gamma_{allow} = 0.012$ rad

Find L_{min}

From Eq. (3-3): $\gamma_{max} = \dfrac{r\phi}{L} = \dfrac{d\phi}{2L}$



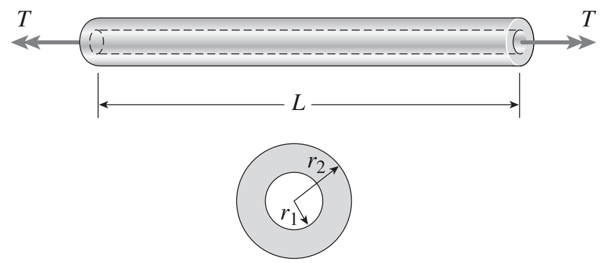
$$L_{min} = \frac{d\phi}{2\gamma_{allow}} = \frac{(50 \text{ mm})(0.08727 \text{ rad})}{(2)(0.012 \text{ rad})}$$

$$L_{min} = 182 \text{ mm} \longleftarrow$$

Problem 3.2-3 A circular aluminum tube subjected to pure torsion by torques T (see figure) has an outer radius r_2 equal to twice the inner radius r_1.

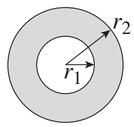
(a) If the maximum shear strain in the tube is measured as 400×10^{-6} rad, what is the shear strain γ_1 at the inner surface?

(b) If the maximum allowable rate of twist is 0.15 degrees per foot and the maximum shear strain is to be kept at 400×10^{-6} rad by adjusting the torque T, what is the minimum required outer radius $(r_2)_{min}$?



Problems 3.2-3, 3.2-4, and 3.2-5

Solution 3.2-3 Circular aluminum tube



$$r_2 = 2r_1$$

$$\gamma_{max} = 400 \times 10^{-6} \text{ rad}$$

$$\theta_{allow} = 0.15°/\text{ft}$$

$$= (0.15°/\text{ft})\left(\frac{\pi}{180}\frac{\text{rad}}{\text{degree}}\right)\left(\frac{1}{12}\frac{\text{ft}}{\text{in.}}\right)$$

$$= 218.2 \times 10^{-6} \text{ rad/in.}$$

(a) Shear strain at inner surface

From Eq. (3-5b):

$$\gamma_1 = \frac{1}{2}\gamma_2 = \frac{1}{2}(400 \times 10^{-6} \text{ rad})$$

$$\gamma_1 = 200 \times 10^{-6} \text{ rad} \longleftarrow$$

(b) Minimum outer radius

From Eq. (3-5a):

$$\gamma_{max} = r_2\frac{\phi}{L} = r_2\theta$$

$$(r_2)_{min} = \frac{\gamma_{max}}{\theta_{allow}} = \frac{400 \times 10^{-6} \text{ rad}}{218.2 \times 10^{-6} \text{ rad/in.}}$$

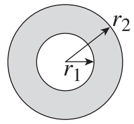
$$(r_2)_{min} = 1.83 \text{ in.} \longleftarrow$$

Problem 3.2-4 A circular steel tube of length $L = 0.90$ m is loaded in torsion by torques T (see figure).

(a) If the inner radius of the tube is $r_1 = 40$ mm and the measured angle of twist between the ends is 0.5°, what is the shear strain γ_1 (in radians) at the inner surface?

(b) If the maximum allowable shear strain is 0.0005 rad and the angle of twist is to be kept at 0.5° by adjusting the torque T, what is the maximum permissible outer radius $(r_2)_{max}$?

Solution 3.2-4 **Circular steel tube**



$L = 0.90$ m

$r_1 = 40$ mm

$$\phi = 0.5° = (0.5°)\left(\frac{\pi}{180}\frac{\text{rad}}{\text{degree}}\right)$$

$$= 0.008727 \text{ rad}$$

$$\gamma_{max} = 0.0005 \text{ rad}$$

(a) SHEAR STRAIN AT INNER SURFACE

From Eq. (3-5b):

$$\gamma_{min} = \gamma_1 = r_1\frac{\phi}{L} = \frac{(40 \text{ mm})(0.008727 \text{ rad})}{900 \text{ mm}}$$

$$\gamma_1 = 388 \times 10^{-6} \text{ rad} \longleftarrow$$

(b) MAXIMUM OUTER RADIUS

From Eq. (3-5a):

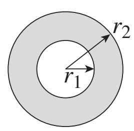
$$\gamma_{max} = \gamma_2 = r_2\frac{\phi}{L}; \quad r_2 = \frac{\gamma_{max}L}{\phi}$$

$$(r_2)_{max} = \frac{(0.0005 \text{ rad})(900 \text{ mm})}{0.008727 \text{ rad}}$$

$$(r_2)_{max} = 51.6 \text{ mm} \longleftarrow$$

Problem 3.2-5 Solve the preceding problem if the length $L = 50$ in., the inner radius $r_1 = 1.5$ in., the angle of twist is 0.6°, and the allowable shear strain is 0.0004 rad.

Solution 3.2-5 **Circular steel tube**



$L = 50$ in.

$r_1 = 1.5$ in.

$$\phi = 0.6° = (0.6°)\left(\frac{\pi}{180}\frac{\text{rad}}{\text{degree}}\right)$$

$$= 0.010472 \text{ rad}$$

$$\gamma_{max} = 0.0004 \text{ rad}$$

(a) SHEAR STRAIN AT INNER SURFACE

From Eq. (3-5b):

$$\gamma_{min} = \gamma_1 = r_1\frac{\phi}{L} = \frac{(1.5 \text{ in.})(0.010472 \text{ rad})}{50 \text{ in.}}$$

$$\gamma_1 = 314 \times 10^{-6} \text{ rad} \longleftarrow$$

(b) MAXIMUM OUTER RADIUS

From Eq. (3-5a):

$$\gamma_{max} = \gamma_2 = r_2\frac{\phi}{L}; \; r_2 = \frac{\gamma_{max}L}{\phi}$$

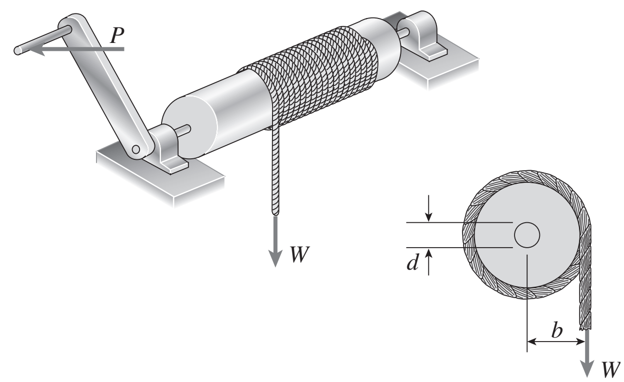
$$(r_2)_{max} = \frac{(0.0004 \text{ rad})(50 \text{ in.})}{0.010472 \text{ rad}}$$

$$(r_2)_{max} = 1.91 \text{ in.} \longleftarrow$$

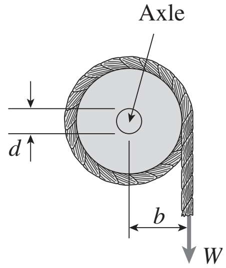
Circular Bars and Tubes

Problem 3.3-1 A prospector uses a hand-powered winch (see figure) to raise a bucket of ore in his mine shaft. The axle of the winch is a steel rod of diameter $d = 0.625$ in. Also, the distance from the center of the axle to the center of the lifting rope is $b = 4.0$ in.

If the weight of the loaded bucket is $W = 100$ lb, what is the maximum shear stress in the axle due to torsion?



Solution 3.3-1 **Hand-powered winch**



$d = 0.625$ in.

$b = 4.0$ in.

$W = 100$ lb

Torque T applied to the axle:

$T = Wb = 400$ lb-in.

MAXIMUM SHEAR STRESS IN THE AXLE

From Eq. (3-12):

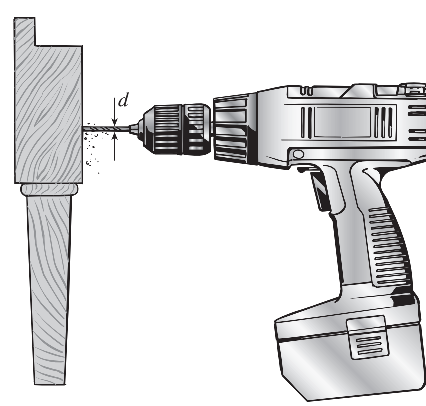
$$\tau_{max} = \frac{16T}{\pi d^3}$$

$$\tau_{max} = \frac{(16)(400 \text{ lb-in})}{\pi(0.625 \text{ in.})^3}$$

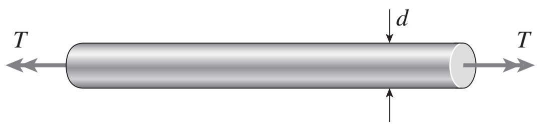
$$\tau_{max} = 8{,}340 \text{ psi} \longleftarrow$$

Problem 3.3-2 When drilling a hole in a table leg, a furniture maker uses a hand-operated drill (see figure) with a bit of diameter $d = 4.0$ mm.

(a) If the resisting torque supplied by the table leg is equal to 0.3 N·m, what is the maximum shear stress in the drill bit?
(b) If the shear modulus of elasticity of the steel is $G = 75$ GPa, what is the rate of twist of the drill bit (degrees per meter)?



Solution 3.3-2 **Torsion of a drill bit**



$d = 4.0$ mm $\quad T = 0.3$ N·m $\quad G = 75$ GPa

MAXIMUM SHEAR STRESS

From Eq. (3-12):

$$\tau_{max} = \frac{16T}{\pi d^3}$$

$$\tau_{max} = \frac{16(0.3\ \text{N}\cdot\text{m})}{\pi(4.0\ \text{mm})^3}$$

$$\tau_{max} = 23.8\ \text{MPa} \longleftarrow$$

RATE OF TWIST

From Eq. (3-14):

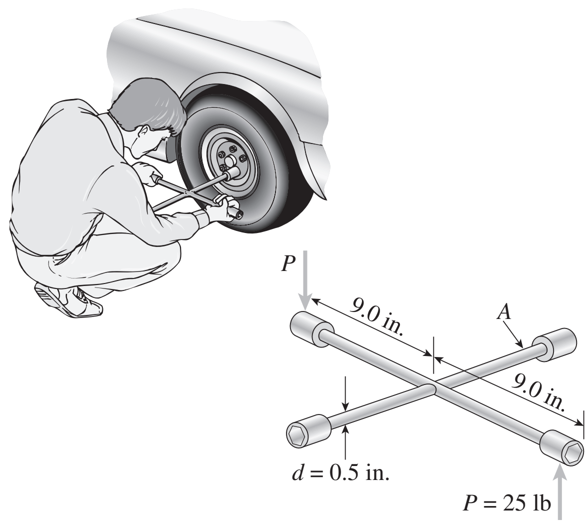
$$\theta = \frac{T}{GI_P}$$

$$\theta = \frac{0.3\ \text{N}\cdot\text{m}}{(75\ \text{GPa})\left(\frac{\pi}{32}\right)(4.0\ \text{mm})^4}$$

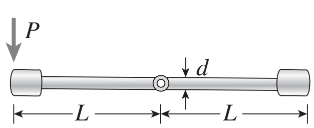
$$\theta = 0.1592\ \text{rad/m} = 9.12°/\text{m} \longleftarrow$$

Problem 3.3-3 While removing a wheel to change a tire, a driver applies forces $P = 25$ lb at the ends of two of the arms of a lug wrench (see figure). The wrench is made of steel with shear modulus of elasticity $G = 11.4 \times 10^6$ psi. Each arm of the wrench is 9.0 in. long and has a solid circular cross section of diameter $d = 0.5$ in.

(a) Determine the maximum shear stress in the arm that is turning the lug nut (arm A).
(b) Determine the angle of twist (in degrees) of this same arm.



Solution 3.3-3 **Lug wrench**

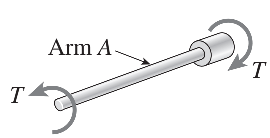


$P = 25$ lb

$L = 9.0$ in.

$d = 0.5$ in.

$G = 11.4 \times 10^6$ psi



$T =$ torque acting on arm A

$T = P(2L) = 2(25\ \text{lb})(9.0\ \text{in.})$

$= 450$ lb-in.

(a) MAXIMUM SHEAR STRESS

From Eq. (3-12):

$$\tau_{max} = \frac{16T}{\pi d^3} = \frac{(16)(450\ \text{lb-in.})}{\pi(0.5\ \text{in.})^3}$$

$$\tau_{max} = 18{,}300\ \text{psi} \longleftarrow$$

(b) ANGLE OF TWIST

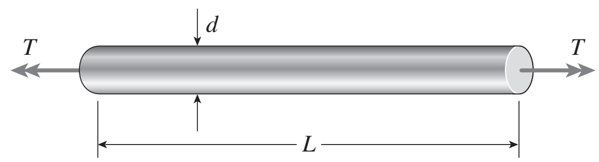
From Eq. (3-15):

$$\phi = \frac{TL}{GI_P} = \frac{(450\ \text{lb-in.})(9.0\ \text{in.})}{(11.4 \times 10^6\ \text{psi})\left(\frac{\pi}{32}\right)(0.5\ \text{in.})^4}$$

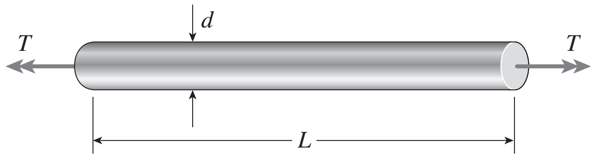
$$\phi = 0.05790\ \text{rad} = 3.32° \longleftarrow$$

Problem 3.3-4 An aluminum bar of solid circular cross section is twisted by torques T acting at the ends (see figure). The dimensions and shear modulus of elasticity are as follows: $L = 1.2$ m, $d = 30$ mm, and $G = 28$ GPa.

(a) Determine the torsional stiffness of the bar.
(b) If the angle of twist of the bar is 4°, what is the maximum shear stress? What is the maximum shear strain (in radians)?



Solution 3.3-4 **Aluminum bar in torsion**



$L = 1.2$ m $\qquad d = 30$ mm

$G = 28$ GPa $\qquad \phi = 4°$

(a) TORSIONAL STIFFNESS

$$k_T = \frac{GI_P}{L} = \frac{G\pi d^4}{32L} = \frac{(28\text{ GPa})(\pi)(30\text{ mm})^4}{32(1.2\text{ m})}$$

$$k_T = 1860\text{ N}\cdot\text{m} \longleftarrow$$

(b) MAXIMUM SHEAR STRESS

$$\phi = 4° = (4°)(\pi/180)rad = 0.069813\text{ rad}$$

From Eq. (3-15):

$$\phi = \frac{TL}{GI_P} \quad T = \frac{GI_P\phi}{L}$$

From Eq. (3-11):

$$\tau_{\max} = \frac{Tr}{I_P} = \frac{Td}{2I_P} = \left(\frac{GI_P\phi}{L}\right)\left(\frac{d}{2I_P}\right)$$

$$\tau_{\max} = \frac{Gd\phi}{2L}$$

$$= \frac{(28\text{ GPa})(30\text{ mm})(0.069813\text{ rad})}{2(1.2\text{ m})}$$

$$= 24.43\text{ MPa}$$

$$\tau_{\max} = 24.4\text{ MPa} \longleftarrow$$

MAXIMUM SHEAR STRAIN

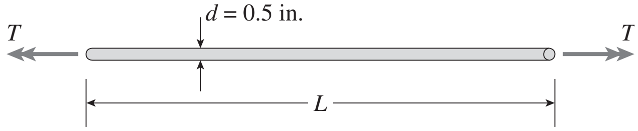
Hooke's Law:

$$\gamma_{\max} = \frac{\tau_{\max}}{G} = \frac{24.43\text{ MPa}}{28\text{ GPa}}$$

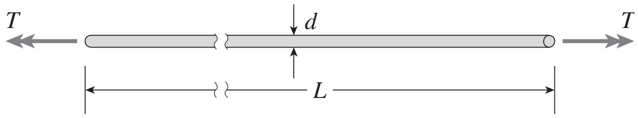
$$\gamma_{\max} = 873 \times 10^{-6}\text{ rad} \longleftarrow$$

Problem 3.3-5 A high-strength steel drill rod used for boring a hole in the earth has a diameter of 0.5 in. (see figure).The allowable shear stress in the steel is 40 ksi and the shear modulus of elasticity is 11,600 ksi.

What is the minimum required length of the rod so that one end of the rod can be twisted 30° with respect to the other end without exceeding the allowable stress?



Solution 3.3-5 Steel drill rod



$G = 11{,}600$ psi

$d = 0.5$ in.

$\phi = 30° = (30°)\left(\frac{\pi}{180}\right)\text{rad} = 0.52360$ rad

$\tau_{\text{allow}} = 40$ ksi

MINIMUM LENGTH

From Eq. (3-12): $\tau_{\max} = \frac{16T}{\pi d^3}$ (1)

From Eq. (3-15): $\phi = \frac{TL}{GI_P} = \frac{32TL}{G\pi d^4}$

$T = \frac{G\pi d^4 \phi}{32L}$; substitute T into Eq. (1):

$$\tau_{\max} = \left(\frac{16}{\pi d^3}\right)\left(\frac{G\pi d^4\phi}{32L}\right) = \frac{Gd\phi}{2L}$$

$$L_{\min} = \frac{Gd\phi}{2\tau_{\text{allow}}}$$

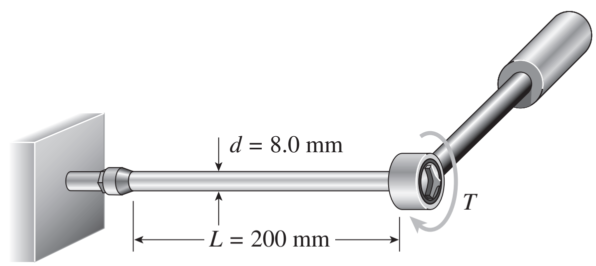
$$= \frac{(11{,}600 \text{ ksi})(0.5 \text{ in.})(0.52360 \text{ rad})}{2(40 \text{ ksi})}$$

$L_{\min} = 38.0$ in. ⟵

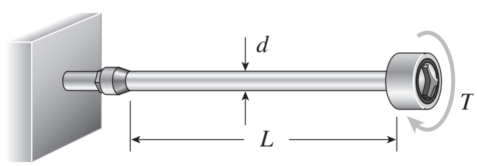
Problem 3.3-6 The steel shaft of a socket wrench has a diameter of 8.0 mm. and a length of 200 mm (see figure).

If the allowable stress in shear is 60 MPa, what is the maximum permissible torque $T_{\max}$ that may be exerted with the wrench?

Through what angle ϕ (in degrees) will the shaft twist under the action of the maximum torque? (Assume $G = 78$ GPa and disregard any bending of the shaft.)



Solution 3.3-6 Socket wrench



$d = 8.0$ mm $\quad L = 200$ mm

$\tau_{\text{allow}} = 60$ MPa $\quad G = 78$ GPa

MAXIMUM PERMISSIBLE TORQUE

From Eq. (3-12): $\tau_{\max} = \frac{16T}{\pi d^3}$

$$T_{\max} = \frac{\pi d^3 \tau_{\max}}{16}$$

$$T_{\max} = \frac{\pi(8.0 \text{ mm})^3(60 \text{ MPa})}{16}$$

$T_{\max} = 6.03$ N · m ⟵

ANGLE OF TWIST

From Eq. (3-15): $\phi = \frac{T_{\max}L}{GI_P}$

From Eq. (3-12): $T_{\max} = \frac{\pi d^3 \tau_{\max}}{16}$

$$\phi = \left(\frac{\pi d^3 \tau_{\max}}{16}\right)\left(\frac{L}{GI_P}\right) \quad I_P = \frac{\pi d^4}{32}$$

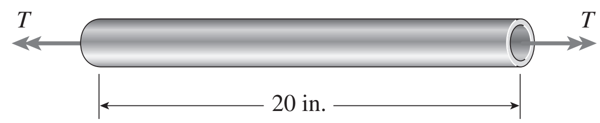
$$\phi = \frac{\pi d^3 \tau_{\max} L(32)}{16G(\pi d^4)} = \frac{2\tau_{\max}L}{Gd}$$

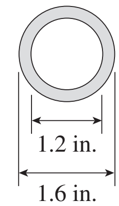
$$\phi = \frac{2(60 \text{ MPa})(200 \text{ mm})}{(78 \text{ GPa})(8.0 \text{ mm})} = 0.03846 \text{ rad}$$

$\phi = (0.03846 \text{ rad})\left(\frac{180}{\pi}\text{deg/rad}\right) = 2.20°$ ⟵

Problem 3.3-7 A circular tube of aluminum is subjected to torsion by torques T applied at the ends (see figure). The bar is 20 in. long, and the inside and outside diameters are 1.2 in. and 1.6 in., respectively. It is determined by measurement that the angle of twist is 3.63° when the torque is 5800 lb-in.

Calculate the maximum shear stress τ_{max} in the tube, the shear modulus of elasticity G, and the maximum shear strain γ_{max} (in radians).



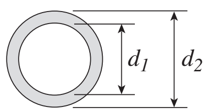


Solution 3.3-7 **Aluminum tube in torsion**

$L = 20$ in.

$d_1 = 1.2$ in.

$d_2 = 1.6$ in.



$T = 5800$ lb-in.

$\phi = 3.63° = 0.063355$ rad

$$I_P = \frac{\pi}{32}(d_2^4 - d_1^4) = 0.43982 \text{ in.}^4$$

MAXIMUM SHEAR STRESS

$$\tau_{max} = \frac{Tr}{I_P} = \frac{(5800 \text{ lb-in.})(0.8 \text{ in.})}{0.43982 \text{ in.}^4}$$

$$\tau_{max} = 10{,}550 \text{ psi} \longleftarrow$$

SHEAR MODULUS OF ELASTICITY

$$\phi = \frac{TL}{GI_P} \qquad G = \frac{TL}{\phi I_P}$$

$$G = \frac{(5800 \text{ lb-in.})(20 \text{ in.})}{(0.063355 \text{ rad})(0.43982 \text{ in.}^4)}$$

$$G = 4.16 \times 10^6 \text{ psi} \longleftarrow$$

MAXIMUM SHEAR STRAIN

$$\gamma_{max} = \frac{\tau_{max}}{G}$$

$$\gamma_{max} = \left(\frac{T_r}{I_P}\right)\left(\frac{\phi I_P}{TL}\right) = \frac{r\phi}{L}$$

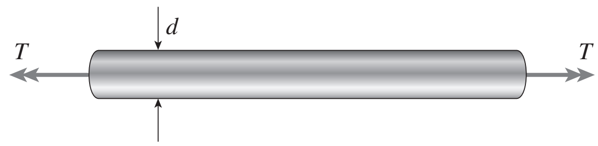
$$\gamma_{max} = \frac{(0.8 \text{ in.})(0.063355 \text{ rad})}{20 \text{ in.}}$$

$$\gamma_{max} = 0.00253 \text{ rad} \longleftarrow$$

Problem 3.3-8 A propeller shaft for a small yacht is made of a solid steel bar 100 mm in diameter. The allowable stress in shear is 50 MPa, and the allowable rate of twist is 2.0° in 3 meters.

Assuming that the shear modulus of elasticity is $G = 80$ GPa, determine the maximum torque T_{max} that can be applied to the shaft.

Solution 3.3-8 **Propeller shaft**



$d = 100$ mm

$G = 80$ GPa $\qquad \tau_{allow} = 50$ MPa

$\theta = 2° \text{ in } 3 \text{ m} = \frac{1}{3}(2°)\left(\frac{\pi}{180}\right) \text{ rad/m}$

$= 0.011636$ rad/m

MAX. TORQUE BASED UPON SHEAR STRESS

$$\tau = \frac{16T}{\pi d^3} \qquad T_1 = \frac{\pi d^3 \tau_{allow}}{16}$$

$$= \frac{\pi(100 \text{ mm})^3(50 \text{ MPa})}{16}$$

$T_1 = 9820 \text{ N} \cdot \text{m} \longleftarrow$

MAX. TORQUE BASED UPON RATE OF TWIST

$$\theta = \frac{T}{GI_P} \qquad T_2 = GI_P\theta = G\left(\frac{\pi d^4}{32}\right)\theta$$

$$= (80 \text{ GPa})\left(\frac{\pi}{32}\right)(100 \text{ mm})^4(0.011636 \text{ rad/m})$$

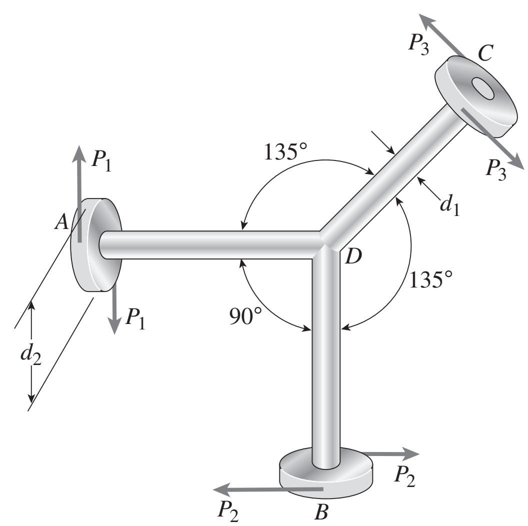
$T_2 = 9140 \text{ N} \cdot \text{m} \longleftarrow$

RATE OF TWIST GOVERNS

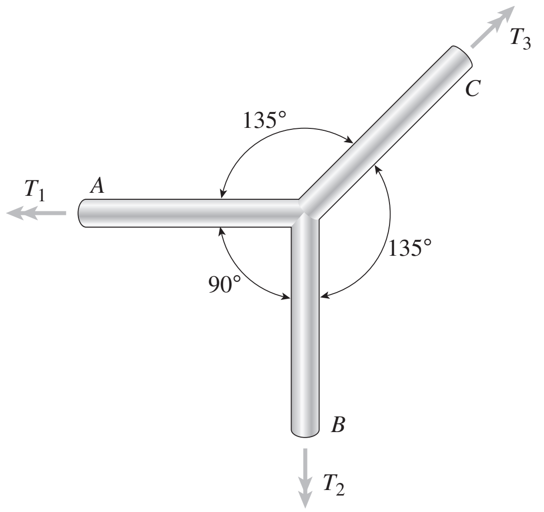
$T_{max} = 9140 \text{ N} \cdot \text{m} \longleftarrow$

Problem 3.3-9 Three identical circular disks *A*, *B*, and *C* are welded to the ends of three identical solid circular bars (see figure). The bars lie in a common plane and the disks lie in planes perpendicular to the axes of the bars. The bars are welded at their intersection *D* to form a rigid connection. Each bar has diameter $d_1 = 0.5$ in. and each disk has diameter $d_2 = 3.0$ in.

Forces P_1, P_2, and P_3 act on disks *A*, *B*, and *C*, respectively, thus subjecting the bars to torsion. If $P_1 = 28$ lb, what is the maximum shear stress τ_{max} in any of the three bars?



Solution 3.3-9 **Three circular bars**



d_1 = diameter of bars

= 0.5 in.

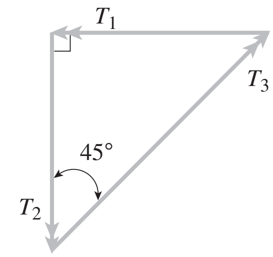
d_2 = diameter of disks

= 3.0 in.

$P_1 = 28$ lb

$T_1 = P_1 d_2 \qquad T_2 = P_2 d_2 \qquad T_3 = P_3 d_2$

The three torques must be in equilibrium



T_3 is the largest torque

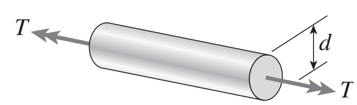
$T_3 = T_1\sqrt{2} = P_1 d_2 \sqrt{2}$

Maximum shear stress (Eq. 3-12)

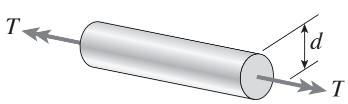
$$\tau_{\max} = \frac{16T}{\pi d^3} = \frac{16T_3}{\pi d_1^3} = \frac{16P_1 d_2\sqrt{2}}{\pi d_1^3}$$

$$\tau_{\max} = \frac{16(28 \text{ lb})(3.0 \text{ in.})\sqrt{2}}{\pi(0.5 \text{ in.})^3} = 4840 \text{ psi} \longleftarrow$$

Problem 3.3-10 The steel axle of a large winch on an ocean liner is subjected to a torque of 1.5 kN·m (see figure). What is the minimum required diameter $d_{\min}$ if the allowable shear stress is 50 MPa and the allowable rate of twist is 0.8°/m? (Assume that the shear modulus of elasticity is 80 GPa.)



Solution 3.3-10 **Axle of a large winch**



$T = 1.5 \text{ kN} \cdot \text{m}$

$G = 80$ GPa

$\tau_{\text{allow}} = 50$ MPa

$$\theta_{\text{allow}} = 0.8°/\text{m} = (0.8°)\left(\frac{\pi}{180}\right)\text{rad/m}$$

$$= 0.013963 \text{ rad/m}$$

Min. diameter based upon shear stress

$$\tau = \frac{16T}{\pi d^3} \qquad d^3 = \frac{16T}{\pi \tau_{\text{allow}}}$$

$$d^3 = \frac{16(1.5 \text{ k}N \cdot \text{m})}{\pi(50 \text{ MPa})} = 152.789 \times 10^{-6} \text{ m}^3$$

$d = 0.05346$ m $\quad d_{\min} = 53.5$ mm

Min. diameter based upon rate of twist

$$\theta = \frac{T}{GI_p} = \frac{32T}{G\pi d^4} \qquad d^4 = \frac{32T}{\pi G \theta_{\text{allow}}}$$

$$d^4 = \frac{32(1.5 \text{ k}N \cdot \text{m})}{\pi(80 \text{ GPa})(0.013963 \text{ rad/m})}$$

$$= 0.00001368 \text{ m}^4$$

$d = 0.0608$ m $\quad d_{\min} = 60.8$ mm

Rate of twist governs

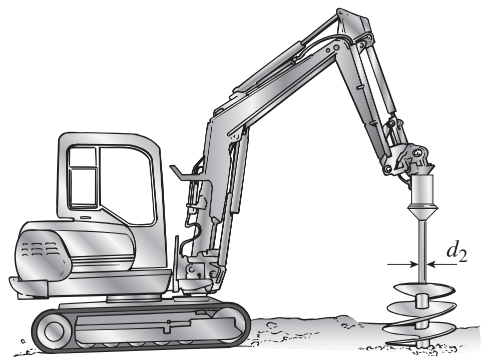
$d_{\min} = 60.8 \text{ mm} \longleftarrow$

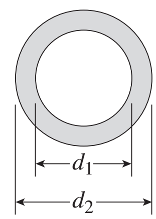
Problem 3.3-11 A hollow steel shaft used in a construction auger has outer diameter $d_2 = 6.0$ in. and inner diameter $d_1 = 4.5$ in. (see figure). The steel has shear modulus of elasticity $G = 11.0 \times 10^6$ psi.

For an applied torque of 150 k-in., determine the following quantities:

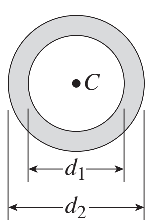
(a) shear stress τ_2 at the outer surface of the shaft,
(b) shear stress τ_1 at the inner surface, and
(c) rate of twist θ (degrees per unit of length).

Also, draw a diagram showing how the shear stresses vary in magnitude along a radial line in the cross section.





Solution 3.3-11 Construction auger



$d_2 = 6.0$ in. $\quad r_2 = 3.0$ in.

$d_1 = 4.5$ in. $\quad r_1 = 2.25$ in.

$G = 11 \times 10^6$ psi

$T = 150$ k-in.

$$I_P = \frac{\pi}{32}(d_2^4 - d_1^4) = 86.98 \text{ in.}^4$$

(a) SHEAR STRESS AT OUTER SURFACE

$$\tau_2 = \frac{Tr_2}{I_P} = \frac{(150\ k\text{-in.})(3.0\text{ in.})}{86.98\text{ in.}^4}$$

$$= 5170 \text{ psi} \quad \longleftarrow$$

(b) SHEAR STRESS AT INNER SURFACE

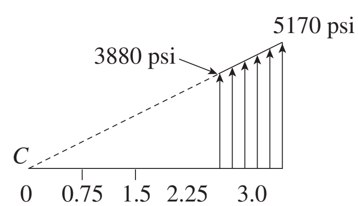
$$\tau_1 = \frac{Tr_1}{I_P} = \frac{r_1}{r_2}\tau_2 = 3880 \text{ psi} \quad \longleftarrow$$

(c) RATE OF TWIST

$$\theta = \frac{T}{GI_P} = \frac{(150\ k\text{-in.})}{(11 \times 10^6 \text{ psi})(86.98 \text{ in.}^4)}$$

$$\theta = 157 \times 10^{-6} \text{ rad/in.} = 0.00898°\text{/in.} \quad \longleftarrow$$

(d) SHEAR STRESS DIAGRAM



Problem 3.3-12 Solve the preceding problem if the shaft has outer diameter $d_2 = 150$ mm and inner diameter $d_1 = 100$ mm. Also, the steel has shear modulus of elasticity $G = 75$ GPa and the applied torque is 16 kN·m.

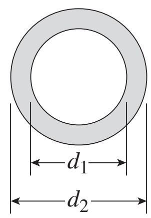
Solution 3.3-12 **Construction auger**

$d_2 = 150$ mm $\quad r_2 = 75$ mm

$d_1 = 100$ mm $\quad r_1 = 50$ mm

$G = 75$ GPa

$T = 16$ kN · m



$$I_P = \frac{\pi}{32}(d_2^4 - d_1^4) = 39.88 \times 10^6 \text{ mm}^4$$

(a) Shear stress at outer surface

$$\tau_2 = \frac{Tr_2}{I_P} = \frac{(16 \text{ k}N \cdot \text{m})(75 \text{ mm})}{39.88 \times 10^6 \text{ mm}^4}$$

$$= 30.1 \text{ MPa} \quad \longleftarrow$$

(b) Shear stress at inner surface

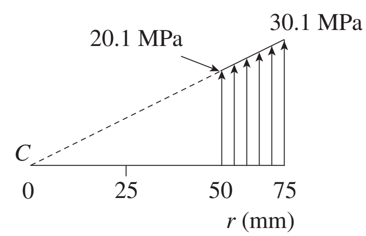
$$\tau_1 = \frac{Tr_1}{I_P} = \frac{r_1}{r_2} \quad \tau_2 = 20.1 \text{ MPa} \quad \longleftarrow$$

(c) Rate of twist

$$\theta = \frac{T}{GI_P} = \frac{16 \text{ k}N \cdot \text{m}}{(75 \text{ GPa})(39.88 \times 10^6 \text{ mm}^4)}$$

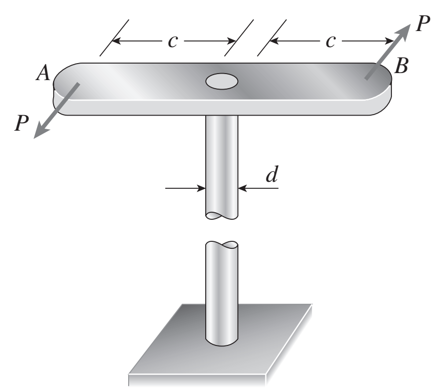
$$\theta = 0.005349 \text{ rad/m} = 0.306°/\text{m} \quad \longleftarrow$$

(d) Shear stress diagram

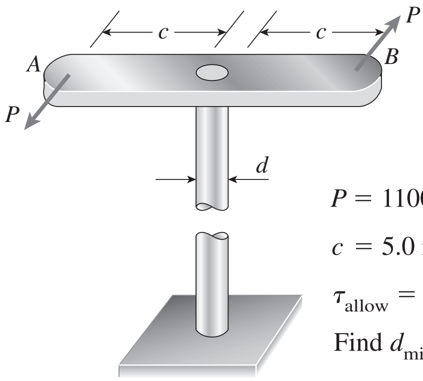


Problem 3.3-13 A vertical pole of solid circular cross section is twisted by horizontal forces $P = 1100$ lb acting at the ends of a horizontal arm AB (see figure). The distance from the outside of the pole to the line of action of each force is $c = 5.0$ in.

If the allowable shear stress in the pole is 4500 psi, what is the minimum required diameter d_{min} of the pole?



Solution 3.3-13 **Vertical pole**



$P = 1100$ lb

$c = 5.0$ in.

$\tau_{allow} = 4500$ psi

Find d_{min}

TORSION FORMULA

$$\tau_{max} = \frac{Tr}{I_P} = \frac{Td}{2I_P}$$

$$T = P(2c + d) \quad I_P = \frac{\pi d^4}{32}$$

$$\tau_{max} = \frac{P(2c + d)d}{\pi d^4/16} = \frac{16P(2c + d)}{\pi d^3}$$

$$(\pi\tau_{max})d^3 - (16P)d - 32Pc = 0$$

SUBSTITUTE NUMERICAL VALUES:

UNITS: Pounds, Inches

$$(\pi)(4500)d^3 - (16)(1100)d - 32(1100)(5.0) = 0$$

or

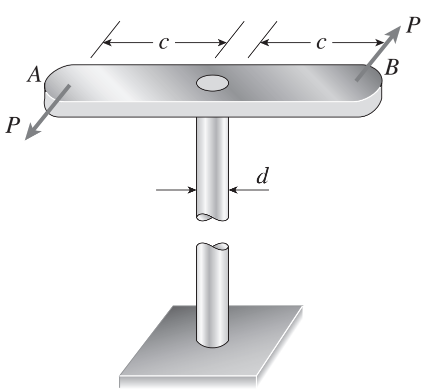
$$d^3 - 1.24495d - 12.4495 = 0$$

Solve numerically: $d = 2.496$ in.

$d_{min} = 2.50$ in. $\longleftarrow$

Problem 3.3-14 Solve the preceding problem if the horizontal forces have magnitude $P = 5.0$ kN, the distance $c = 125$ mm, and the allowable shear stress is 30 MPa.

Solution 3.3-14 **Vertical pole**



$P = 5.0$ kN

$c = 125$ mm

$\tau_{allow} = 30$ MPa

Find d_{min}

TORSION FORMULA

$$\tau_{max} = \frac{Tr}{I_P} = \frac{Td}{2I_P}$$

$$T = P(2c + d) \quad I_P = \frac{\pi d^4}{32}$$

$$\tau_{max} = \frac{P(2c + d)d}{\pi d^4/16} = \frac{16P(2c + d)}{\pi d^3}$$

$$(\pi\tau_{max})d^3 - (16P)d - 32Pc = 0$$

SUBSTITUTE NUMERICAL VALUES:

UNITS: Newtons, Meters

$$(\pi)(30 \times 10^6)d^3 - (16)(5000)d - 32(5000)(0.125) = 0$$

or

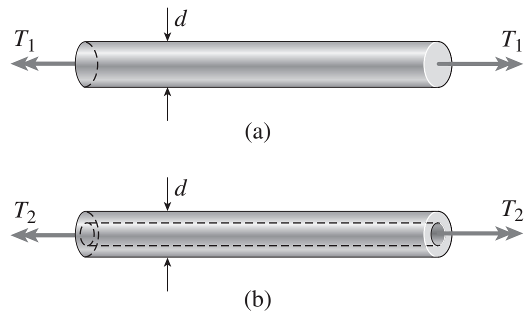
$$d^3 - 848.826 \times 10^{-6}d - 212.207 \times 10^{-6} = 0$$

Solve numerically: $d = 0.06438$ m

$d_{min} = 64.4$ mm $\longleftarrow$

Problem 3.3-15 A solid brass bar of diameter $d = 1.2$ in. is subjected to torques T_1, as shown in part (a) of the figure. The allowable shear stress in the brass is 12 ksi.

(a) What is the maximum permissible value of the torques T_1?
(b) If a hole of diameter 0.6 in. is drilled longitudinally through the bar, as shown in part (b) of the figure, what is the maximum permissible value of the torques T_2?
(c) What is the percent decrease in torque and the percent decrease in weight due to the hole?

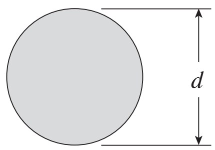


Solution 3.3-15 Brass bar in torsion

(a) SOLID BAR

$d = 1.2$ in.

$\tau_{\text{allow}} = 12$ ksi

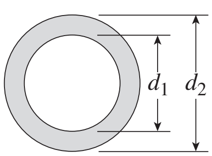


Find max. torque T_1

$$\tau_{\max} = \frac{16T}{\pi d^3} \qquad T_1 = \frac{\pi d^3 \tau_{\text{allow}}}{16}$$

$$T_1 = \frac{\pi(1.2 \text{ in.})^3(12 \text{ ksi})}{16}$$

$$= 4072 \text{ lb-in.} \longleftarrow$$



(b) BAR WITH A HOLE

$d_2 = d = 1.2$ in.

$d_1 = 0.6$ in.

$$\tau_{\max} = \frac{Tr}{I_P} = \frac{Td/2}{\frac{\pi}{32}(d_2^4 - d_1^4)} = \frac{16Td_2}{\pi(d_2^4 - d_1^4)}$$

$$T_2 = \frac{\pi(d_2^4 - d_1^4)\tau_{\text{allow}}}{16d_2}$$

$$T_2 = \frac{\pi[(1.2 \text{ in.})^4 - (0.6 \text{ in.})^4](12 \text{ ksi})}{16(1.2 \text{ in.})}$$

$$T_2 = 3817 \text{ lb-in.} \longleftarrow$$

(c) PERCENT DECREASE IN TORQUE

$$\frac{T_2}{T_1} = \frac{\pi(d_2^4 - d_1^4)\tau_{\text{allow}}}{16d_2} \cdot \frac{16}{\pi d_2^3 \tau_{\text{allow}}} = 1 - \left(\frac{d_1}{d_2}\right)^4$$

$$\frac{d_1}{d_2} = \frac{1}{2} \qquad \frac{T_2}{T_1} = 0.9375$$

% decrease = 6.25% $\longleftarrow$

PERCENT DECREASE IN WEIGHT

$$\frac{W_2}{W_1} = \frac{A_2}{A_1} = \frac{d_2^2 - d_1^2}{d_2^2} = 1 - \left(\frac{d_1}{d_2}\right)^2$$

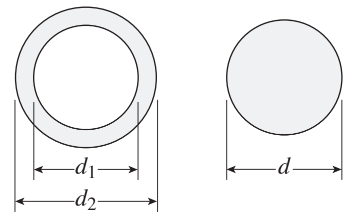
$$\frac{d_1}{d_2} = \frac{1}{2} \qquad \frac{W_2}{W_1} = \frac{3}{4}$$

% decrease = 25% $\longleftarrow$

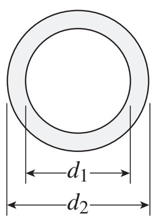
NOTE: The hollow bar weighs 25% less than the solid bar with only a 6.25% decrease in strength.

Problem 3.3-16 A hollow aluminum tube used in a roof structure has an outside diameter $d_2 = 100$ mm and an inside diameter $d_1 = 80$ mm (see figure). The tube is 2.5 m long, and the aluminum has shear modulus $G = 28$ GPa.

(a) If the tube is twisted in pure torsion by torques acting at the ends, what is the angle of twist ϕ (in degrees) when the maximum shear stress is 50 MPa?
(b) What diameter d is required for a solid shaft (see figure) to resist the same torque with the same maximum stress?
(c) What is the ratio of the weight of the hollow tube to the weight of the solid shaft?



Solution 3.3-16 **Hollow aluminum tube**



$d_2 = 100$ mm

$d_1 = 80$ mm

$L = 2.5$ m

$G = 28$ GPa

$\tau_{max} = 50$ MPa

(a) ANGLE OF TWIST FOR THE TUBE

$$\tau_{max} = \frac{Tr}{I_p} = \frac{Td_2}{2I_p}, \quad T = \frac{2I_p\tau_{max}}{d_2}$$

$$\phi = \frac{TL}{GI_p} = \left(\frac{2I_p\tau_{max}}{d_2}\right)\left(\frac{L}{GI_p}\right)$$

$$\phi = \frac{2\tau_{max}L}{Gd_2}$$

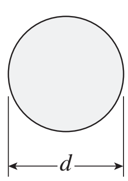
$$\phi = \frac{2(50 \text{ MPa})(2.5 \text{ m})}{(28 \text{ GPa})(100 \text{ mm})} = 0.08929 \text{ rad}$$

$$\phi = 5.12° \quad \longleftarrow$$

(b) DIAMETER OF A SOLID SHAFT

τ_{max} is the same as for tube.

Torque is the same.



For the tube: $T = \frac{2I_P\tau_{max}}{d_2}$

$$T = \frac{2\tau_{max}}{d_2}\left(\frac{\pi}{32}\right)(d_2^4 - d_1^4)$$

FOR THE SOLID SHAFT:

$$\tau_{max} = \frac{16T}{\pi d^3} = \frac{16}{\pi d^3}\left(\frac{2\tau_{max}}{d_2}\right)\left(\frac{\pi}{32}\right)(d_2^4 - d_1^4)$$

Solve for d^3: $d^3 = \frac{d_2^4 - d_1^4}{d_2}$

$$d^3 = \frac{(100 \text{ mm})^4 - (80 \text{ mm})^4}{100 \text{ mm}} = 590{,}400 \text{ mm}^3$$

$$d = 83.9 \text{ mm} \quad \longleftarrow$$

(c) RATIO OF WEIGHTS

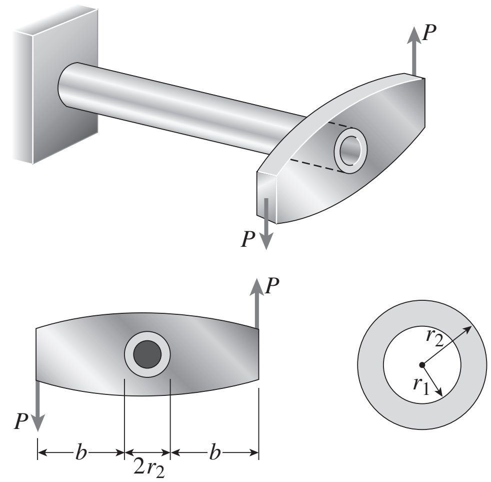
$$\frac{W_{tube}}{W_{solid}} = \frac{A_{tube}}{A_{solid}} = \frac{d_2^2 - d_1^2}{d^2}$$

$$\frac{W_{tube}}{W_{solid}} = \frac{(100 \text{ mm})^2 - (80 \text{ mm})^2}{(83.9 \text{ mm})^2} = 0.51 \quad \longleftarrow$$

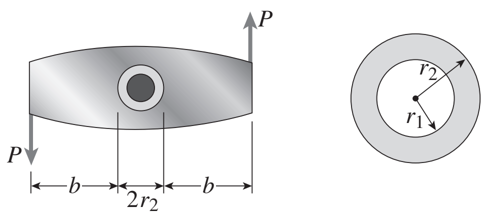
The weight of the tube is 51% of the weight of the solid shaft, but they resist the same torque.

Problem 3.3-17 A circular tube of inner radius r_1 and outer radius r_2 is subjected to a torque produced by forces $P = 900$ lb (see figure). The forces have their lines of action at a distance $b = 5.5$ in. from the outside of the tube.

If the allowable shear stress in the tube is 6300 psi and the inner radius $r_1 = 1.2$ in., what is the minimum permissible outer radius r_2?



Solution 3.3-17 Circular tube in torsion



$P = 900$ lb

$b = 5.5$ in.

$\tau_{\text{allow}} = 6300$ psi

$r_1 = 1.2$ in.

Find minimum permissible radius r_2

TORSION FORMULA

$T = 2P(b + r_2)$

$$I_P = \frac{\pi}{2}(r_2^4 - r_1^4)$$

$$\tau_{\max} = \frac{Tr_2}{I_P} = \frac{2P(b + r_2)r_2}{\frac{\pi}{2}(r_2^4 - r_1^4)} = \frac{4P(b + r_2)r_2}{\pi(r_2^4 - r_1^4)}$$

All terms in this equation are known except r_2.

SOLUTION OF EQUATION

UNITS: Pounds, Inches

Substitute numerical values:

$$6300 \text{ psi} = \frac{4(900 \text{ lb})(5.5 \text{ in.} + r_2)(r_2)}{\pi[(r_2^4) - (1.2 \text{ in.})^4]}$$

or

$$\frac{r_2^4 - 2.07360}{r_2(r_2 + 5.5)} - 0.181891 = 0$$

or

$$r_2^4 - 0.181891\, r_2^2 - 1.000402\, r_2 - 2.07360 = 0$$

Solve numerically:

$r_2 = 1.3988$ in.

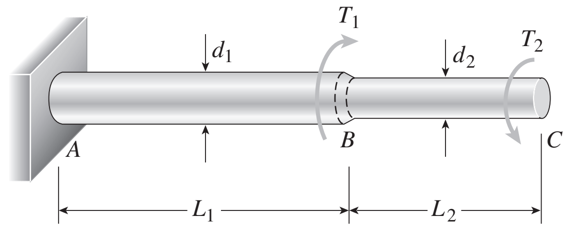
MINIMUM PERMISSIBLE RADIUS

$r_2 = 1.40$ in. $\longleftarrow$

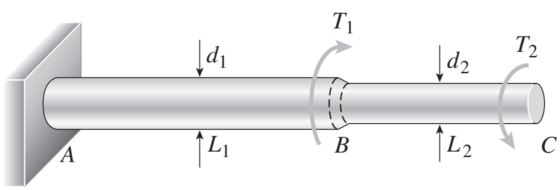
Nonuniform Torsion

Problem 3.4-1 A stepped shaft *ABC* consisting of two solid circular segments is subjected to torques T_1 and T_2 acting in opposite directions, as shown in the figure. The larger segment of the shaft has diameter $d_1 = 2.25$ in. and length $L_1 = 30$ in.; the smaller segment has diameter $d_2 = 1.75$ in. and length $L_2 = 20$ in. The material is steel with shear modulus $G = 11 \times 10^6$ psi, and the torques are $T_1 = 20{,}000$ lb-in. and $T_2 = 8{,}000$ lb-in.

Calculate the following quantities: (a) the maximum shear stress τ_{max} in the shaft, and (b) the angle of twist ϕ_C (in degrees) at end *C*.



Solution 3.4-1 **Stepped shaft**



$d_1 = 2.25$ in. $\quad L_1 = 30$ in.

$d_2 = 1.75$ in. $\quad L_2 = 20$ in.

$G = 11 \times 10^6$ psi

$T_1 = 20{,}000$ lb-in.

$T_2 = 8{,}000$ lb-in.

SEGMENT *AB*

$T_{AB} = T_2 - T_1 = -12{,}000$ lb-in.

$$\tau_{AB} = \left|\frac{16\,T_{AB}}{\pi d_1^3}\right| = \frac{16(12{,}000 \text{ lb-in.})}{\pi(2.25 \text{ in.})^3} = 5365 \text{ psi}$$

$$\phi_{AB} = \frac{T_{AB}L_1}{G(I_p)_{AB}} = \frac{(-12{,}000 \text{ lb-in.})(30 \text{ in.})}{(11 \times 10^6 \text{ psi})\left(\frac{\pi}{32}\right)(2.25 \text{ in.})^4}$$

$$= -0.013007 \text{ rad}$$

SEGMENT *BC*

$T_{BC} = +T_2 = 8{,}000$ lb-in.

$$\tau_{BC} = \frac{16\,T_{BC}}{\pi d_2^3} = \frac{16(8{,}000 \text{ lb-in.})}{\pi(1.75 \text{ in.})^3} = 7602 \text{ psi}$$

$$\phi_{BC} = \frac{T_{BC}L_2}{G(I_p)_{BC}} = \frac{(8{,}000 \text{ lb-in.})(20 \text{ in.})}{(11 \times 10^6 \text{ psi})\left(\frac{\pi}{32}\right)(1.75 \text{ in.})^4}$$

$$= +0.015797 \text{ rad}$$

(a) MAXIMUM SHEAR STRESS

Segment BC has the maximum stress

$\tau_{max} = 7600$ psi $\longleftarrow$

(b) ANGLE OF TWIST AT END *C*

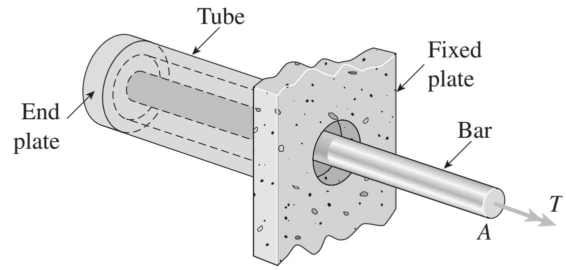
$\phi_C = \phi_{AB} + \phi_{BC} = (-0.013007 + 0.015797)$ rad

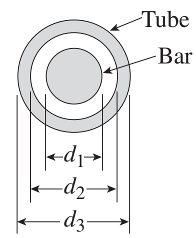
$\phi_C = 0.002790 \quad$ rad $= 0.16°$ $\longleftarrow$

Problem 3.4-2 A circular tube of outer diameter $d_3 = 70$ mm and inner diameter $d_2 = 60$ mm is welded at the right-hand end to a fixed plate and at the left-hand end to a rigid end plate (see figure). A solid circular bar of diameter $d_1 = 40$ mm is inside of, and concentric with, the tube. The bar passes through a hole in the fixed plate and is welded to the rigid end plate.

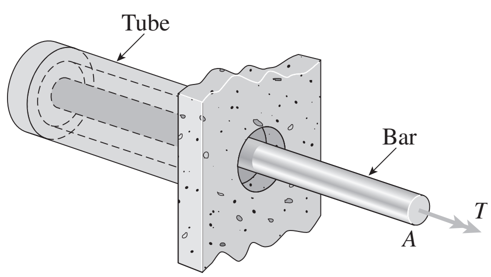
The bar is 1.0 m long and the tube is half as long as the bar. A torque $T = 1000$ N·m acts at end A of the bar. Also, both the bar and tube are made of an aluminum alloy with shear modulus of elasticity $G = 27$ GPa.

(a) Determine the maximum shear stresses in both the bar and tube.
(b) Determine the angle of twist (in degrees) at end A of the bar.





Solution 3.4-2 **Bar and tube**



TUBE

$d_3 = 70$ mm $\quad d_2 = 60$ mm

$L_{tube} = 0.5$ m $\quad G = 27$ GPa

$$(I_p)_{tube} = \frac{\pi}{32}(d_3^4 - d_2^4)$$

$$= 1.0848 \times 10^6 \text{ mm}^4$$

BAR

$d_1 = 40$ mm $\quad L_{bar} = 1.0$ m $\quad G = 27$ GPa

$$(I_p)_{bar} = \frac{\pi d_1^4}{32} = 251.3 \times 10^3 \text{ mm}^4$$

TORQUE

$T = 1000 \text{ N} \cdot \text{m}$

(a) MAXIMUM SHEAR STRESSES

$$\text{Bar: } \tau_{bar} = \frac{16T}{\pi d_1^3} = 79.6 \text{ MPa} \quad \longleftarrow$$

$$\text{Tube: } \tau_{tube} = \frac{T(d_3/2)}{(I_p)_{tube}} = 32.3 \text{ MPa} \quad \longleftarrow$$

(b) ANGLE OF TWIST AT END A

$$\text{Bar: } \phi_{bar} = \frac{TL_{bar}}{G(I_p)_{bar}} = 0.1474 \text{ rad}$$

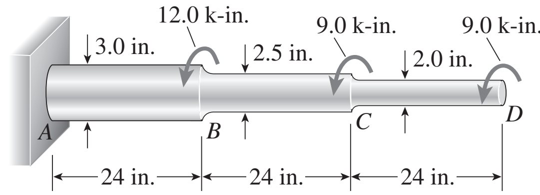
$$\text{Tube: } \phi_{tube} = \frac{TL_{tube}}{G(I_p)_{tube}} = 0.0171 \text{ rad}$$

$$\phi_A = \phi_{bar} + \phi_{tube} = 0.1474 + 0.0171 = 0.1645 \text{ rad}$$

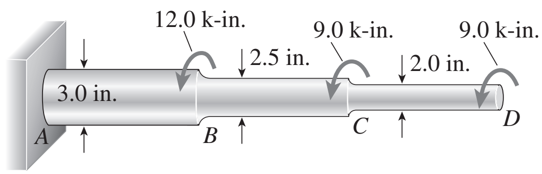
$$\phi_A = 9.43° \quad \longleftarrow$$

Problem 3.4-3 A stepped shaft *ABCD* consisting of solid circular segments is subjected to three torques, as shown in the figure. The torques have magnitudes 12.0 k-in., 9.0 k-in., and 9.0 k-in. The length of each segment is 24 in. and the diameters of the segments are 3.0 in., 2.5 in., and 2.0 in. The material is steel with shear modulus of elasticity $G = 11.6 \times 10^3$ ksi.

(a) Calculate the maximum shear stress τ_{max} in the shaft.
(b) Calculate the angle of twist ϕ_D (in degrees) at end *D*.



Solution 3.4-3 **Stepped shaft**



$G = 11.6 \times 10^3$ ksi

$r_{AB} = 1.5$ in.

$r_{BC} = 1.25$ in. $\quad r_{CD} = 1.0$ in.

$L_{AB} = L_{BC} = L_{CD} = 24$ in.

TORQUES

$T_{AB} = 12.0 + 9.0 + 9.0 = 30$ k-in.

$T_{BC} = 9.0 + 9.0 = 18$ k-in.

$T_{CD} = 9.0$ k-in.

POLAR MOMENTS OF INERTIA

$$(I_p)_{AB} = \frac{\pi}{32}(3.0 \text{ in.})^4 = 7.952 \text{ in.}^4$$

$$(I_p)_{BC} = \frac{\pi}{32}(2.5 \text{ in.})^4 = 3.835 \text{ in.}^4$$

$$(I_p)_{CD} = \frac{\pi}{32}(2.0 \text{ in.})^4 = 1.571 \text{ in.}^4$$

(a) SHEAR STRESSES

$$\tau_{AB} = \frac{T_{AB}\, r_{AB}}{(I_p)_{AB}} = 5660 \text{ psi}$$

$$\tau_{BC} = \frac{T_{BC}\, r_{BC}}{(I_p)_{BC}} = 5870 \text{ psi}$$

$$\tau_{CD} = \frac{T_{CD}\, r_{CD}}{(I_p)_{CD}} = 5730 \text{ psi}$$

$\tau_{max} = 5870$ psi $\longleftarrow$

(b) ANGLE OF TWIST AT END *D*

$$\phi_{AB} = \frac{T_{AB} L_{AB}}{G(I_p)_{AB}} = 0.007805 \text{ rad}$$

$$\phi_{BC} = \frac{T_{BC} L_{BC}}{G(I_p)_{BC}} = 0.009711 \text{ rad}$$

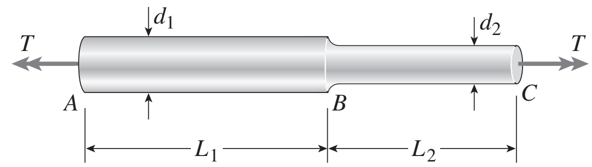
$$\phi_{CD} = \frac{T_{CD} L_{CD}}{G(I_p)_{CD}} = 0.011853 \text{ rad}$$

$\phi_D = \phi_{AB} + \phi_{BC} + \phi_{CD} = 0.02937$ rad

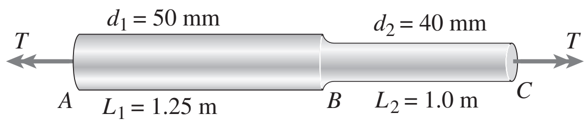
$\phi_D = 1.68°$ $\longleftarrow$

Problem 3.4-4 A solid circular bar ABC consists of two segments, as shown in the figure. One segment has diameter $d_1 = 50$ mm and length $L_1 = 1.25$ m; the other segment has diameter $d_2 = 40$ mm and length $L_2 = 1.0$ m.

What is the allowable torque T_{allow} if the shear stress is not to exceed 30 MPa and the angle of twist between the ends of the bar is not to exceed 1.5°? (Assume $G = 80$ GPa.)



Solutions 3.4-4 Bar consisting of two segments



$\tau_{allow} = 30$ MPa

$\phi_{allow} = 1.5° = 0.02618$ rad

$G = 80$ GPa

Allowable torque based upon shear stress

Segment BC has the smaller diameter and hence the larger stress.

$$\tau_{max} = \frac{16T}{\pi d^3} \qquad T_{allow} = \frac{\pi d_2^3 \tau_{allow}}{16} = 3.77 \text{ N} \cdot \text{m}$$

Allowable torque based upon angle of twist

$$\phi = \sum \frac{T_i L_i}{G I_{Pi}} = \frac{TL_1}{GI_{P1}} + \frac{TL_2}{GI_{P2}} = \frac{T}{G}\left(\frac{L_1}{I_{P1}} + \frac{L_2}{I_{P2}}\right)$$

$$\phi = \frac{32T}{\pi G}\left(\frac{L_1}{d_1^4} + \frac{L_2}{d_2^4}\right)$$

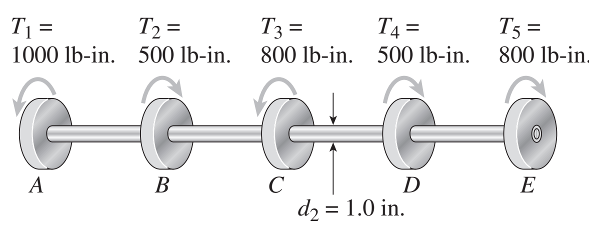
$$T_{allow} = \frac{\pi \phi_{allow} G}{32\left(\frac{L_4}{d_1^1} + \frac{L_4}{d_2^2}\right)} = 348 \text{ N} \cdot \text{m}$$

Angle of twist governs

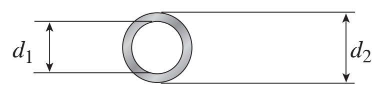
$T_{allow} = 348$ N · m ⟵

Problem 3.4-5 A hollow tube $ABCDE$ constructed of monel metal is subjected to five torques acting in the directions shown in the figure. The magnitudes of the torques are $T_1 = 1000$ lb-in., $T_2 = T_4 = 500$ lb-in., and $T_3 = T_5 = 800$ lb-in. The tube has an outside diameter $d_2 = 1.0$ in. The allowable shear stress is 12,000 psi and the allowable rate of twist is 2.0°/ft.

Determine the maximum permissible inside diameter d_1 of the tube.



Solution 3.4-5 **Hollow tube of monel metal**



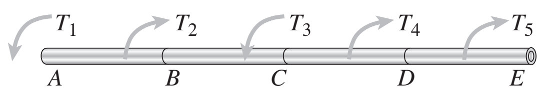
$d_2 = 1.0$ in. $\tau_{allow} = 12{,}000$ psi

$\theta_{allow} = 2°/\text{ft} = 0.16667°/\text{in.}$

$= 0.002909$ rad/in.

From Table H-2, Appendix H: $G = 9500$ ksi

TORQUES



$T_1 = 1000$ lb-in. $T_2 = 500$ lb-in. $T_3 = 800$ lb-in.

$T_4 = 500$ lb-in. $T_5 = 800$ lb-in.

INTERNAL TORQUES

$T_{AB} = -T_1 = -1000$ lb-in.

$T_{BC} = -T_1 + T_2 = -500$ lb-in.

$T_{CD} = -T_1 + T_2 - T_3 = -1300$ lb-in.

$T_{DE} = -T_1 + T_2 - T_3 + T_4 = -800$ lb-in.

Largest torque (absolute value only):

$T_{max} = 1300$ lb-in.

REQUIRED POLAR MOMENT OF INERTIA BASED UPON ALLOWABLE SHEAR STRESS

$$\tau_{max} = \frac{T_{max} r}{I_P} \qquad I_P = \frac{T_{max}(d_2/2)}{\tau_{allow}} = 0.05417 \text{ in.}^4$$

REQUIRED POLAR MOMENT OF INERTIA BASED UPON ALLOWABLE ANGLE OF TWIST

$$\theta = \frac{T_{max}}{GI_P} \qquad I_P = \frac{T_{max}}{G\theta_{allow}} = 0.04704 \text{ in.}^4$$

SHEAR STRESS GOVERNS

Required $I_P = 0.05417$ in.^{4}

$$I_P = \frac{\pi}{32}(d_2^4 - d_1^4)$$

$$d_1^4 = d_2^4 - \frac{32 I_P}{\pi} = (1.0 \text{ in.})^4 - \frac{32(0.05417 \text{ in.}^4)}{\pi}$$

$$= 0.4482 \text{ in.}^4$$

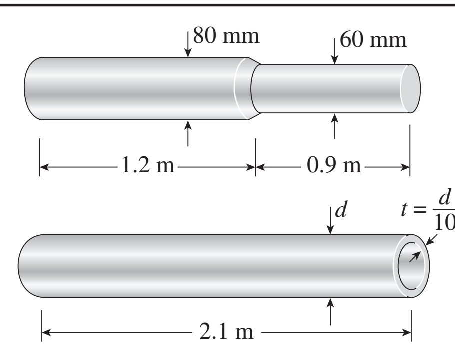
$d_1 = 0.818$ in. $\longleftarrow$

(Maximum permissible inside diameter)

Problem 3.4-6 A shaft of solid circular cross section consisting of two segments is shown in the first part of the figure. The left-hand segment has diameter 80 mm and length 1.2 m; the right-hand segment has diameter 60 mm and length 0.9 m.

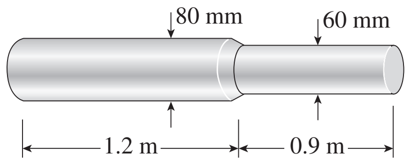
Shown in the second part of the figure is a hollow shaft made of the same material and having the same length. The thickness t of the hollow shaft is $d/10$, where d is the outer diameter. Both shafts are subjected to the same torque.

If the hollow shaft is to have the same torsional stiffness as the solid shaft, what should be its outer diameter d?



Solution 3.4-6 Solid and hollow shafts

SOLID SHAFT CONSISTING OF TWO SEGMENTS

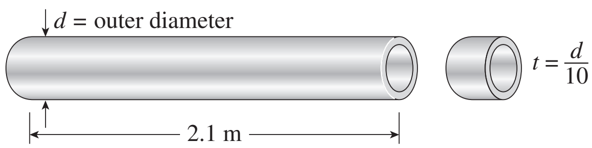


$$\phi_1 = \Sigma \frac{TL_i}{GI_{Pi}} = \frac{T(1.2\text{ m})}{G\left(\frac{\pi}{32}\right)(80\text{ mm})^4} + \frac{T(0.9\text{ m})}{G\left(\frac{\pi}{32}\right)(60\text{ mm})^4}$$

$$= \frac{32T}{\pi G}(29{,}297\text{ m}^{-3} + 69{,}444\text{ m}^{-3})$$

$$= \frac{32T}{\pi G}(98{,}741\text{ m}^{-3})$$

HOLLOW SHAFT



d_0 = inner diameter = $0.8d$

$$\phi_2 = \frac{TL}{GI_p} = \frac{T(2.1\text{ m})}{G\left(\frac{\pi}{32}\right)[d^4 - (0.8d)^4]}$$

$$= \frac{32T}{\pi G}\left(\frac{2.1\text{ m}}{0.5904\, d^4}\right) = \frac{32T}{\pi G}\left(\frac{3.5569\text{ m}}{d^4}\right)$$

UNITS: d = meters

TORSIONAL STIFFNESS

$k_T = \frac{T}{\phi}$ Torque T is the same for both shafts.

$\therefore$ For equal stiffnesses, $\phi_1 = \phi_2$

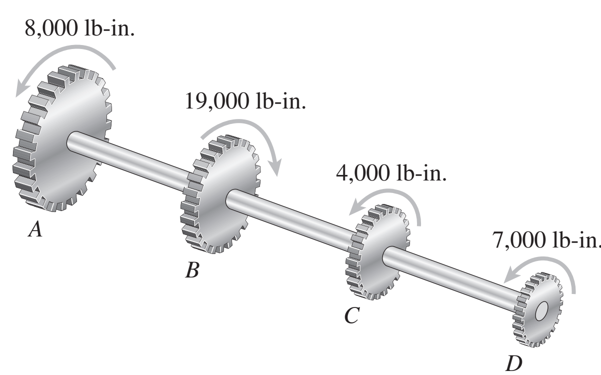
$$98{,}741\text{ m}^{-3} = \frac{3.5569\text{ m}}{d^4}$$

$$d^4 = \frac{3.5569}{98{,}741} = 36.023 \times 10^{-6}\text{ m}^4$$

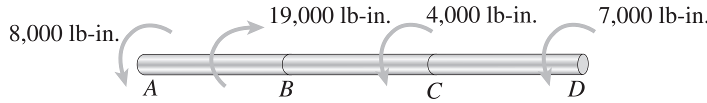
$d = 0.0775\text{ m} = 77.5\text{ mm}$ ⟵

Problem 3.4-7 Four gears are attached to a circular shaft and transmit the torques shown in the figure. The allowable shear stress in the shaft is 10,000 psi.

(a) What is the required diameter d of the shaft if it has a solid cross section?
(b) What is the required outside diameter d if the shaft is hollow with an inside diameter of 1.0 in.?



Solutions 3.4-7 Shaft with four gears



$$\tau_{\text{allow}} = 10{,}000 \text{ psi} \qquad T_{BC} = +11{,}000 \text{ lb-in.}$$

$$T_{AB} = -8000 \text{ lb-in.} \qquad T_{CD} = +7000 \text{ lb-in.}$$

(a) Solid shaft

$$\tau_{\max} = \frac{16T}{\pi d^3}$$

$$d^3 = \frac{16T_{\max}}{\pi\tau_{\text{allow}}} = \frac{16(11{,}000 \text{ lb-in.})}{\pi(10{,}000 \text{ psi})} = 5.602 \text{ in.}^3$$

Required $d = 1.78$ in. $\longleftarrow$

(b) Hollow shaft

Inside diameter $d_0 = 1.0$ in.

$$\tau_{\max} = \frac{Tr}{I_p} \qquad \tau_{\text{allow}} = \frac{T_{\max}\left(\frac{d}{2}\right)}{I_p}$$

$$10{,}000 \text{ psi} = \frac{(11{,}000 \text{ lb-in.})\left(\frac{d}{2}\right)}{\left(\frac{\pi}{32}\right)[d^4 - (1.0 \text{ in.})^4]}$$

Units: d = inches

$$10{,}000 = \frac{56{,}023\, d}{d^4 - 1}$$

or

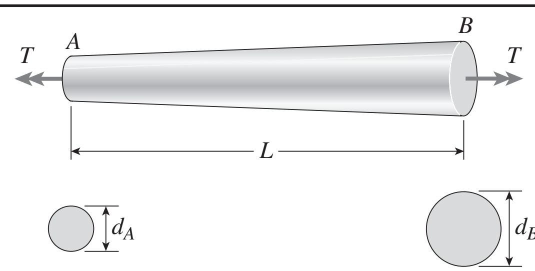
$$d_4 - 5.6023\, d - 1 = 0$$

Solving, $d = 1.832$

Required $d = 1.83$ in. $\longleftarrow$

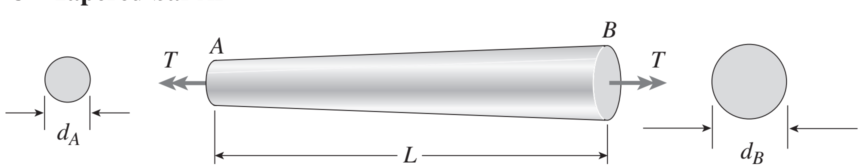
Problem 3.4-8 A tapered bar AB of solid circular cross section is twisted by torques T (see figure). The diameter of the bar varies linearly from d_A at the left-hand end to d_B at the right-hand end.

For what ratio d_B/d_A will the angle of twist of the tapered bar be one-half the angle of twist of a prismatic bar of diameter d_A? (The prismatic bar is made of the same material, has the same length, and is subjected to the same torque as the tapered bar.) *Hint*: Use the results of Example 3-5.



Problems 3.4-9, 3.2-4, and 3.4-10

Solution 3.4-8 Tapered bar *AB*



Tapered bar (From Eq. 3-27)

$$\phi_1 = \frac{TL}{G(I_P)_A}\left(\frac{\beta^2 + \beta + 1}{3\beta^3}\right) \qquad \beta = \frac{d_B}{d_A}$$

Prismatic bar

$$\phi_2 = \frac{TL}{G(I_P)_A}$$

Angle of twist

$$\phi_1 = \frac{1}{2}\phi_2 \qquad \frac{\beta^2 + \beta + 1}{3\beta^3} = \frac{1}{2}$$

or $\quad 3\beta^3 - 2\beta^2 - 2\beta - 2 = 0$

Solve numerically:

$$\beta = \frac{d_B}{d_A} = 1.45 \longleftarrow$$